

EFFECT OF SIDE WALLS ON HEAT TRANSFER THROUGH A VERTICAL AIR LAYER IN LAMINAR NATURAL CONVECTION

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Abstract – The paper presents the results of numerical investigation of local and average heat transfer of a vertically bounded air layer, one of the side boundaries of which has finite thickness and thermal conductivity. The effect of the ratio of the wall to air thermal resistances on heat transfer is discussed. The investigation was carried out over the following range of parameters: $Pr = 0.72$; $Gr < 10^7$; $1 \leq h \leq 10$; $0 \leq \alpha \leq 0.14$.

NOMENCLATURE

<p>a, thermal diffusivity of air;</p> <p>Br, $= (k_f/k_s)(\delta/y)Ra_y^{1/4}$, Brun number;</p> <p>$g$, gravity acceleration;</p> <p>Gr, $= g\beta \Delta T L^3/\nu^2$, Grashof number;</p> <p>h, $= H/L$;</p> <p>H, height of cavity;</p> <p>$k_f(k_s)$, thermal conductivity of air (wall);</p> <p>L, width of cavity;</p> <p>n, internal normal to boundary of the cavity;</p> <p>Nu_b, $= (qL/k_f \Delta T) = -(\partial\theta/\partial n) _{x=0}$, local Nusselt number;</p> <p>Nu, $= -\frac{1}{h} \int_0^h (\partial\theta/\partial n) _{x=0} dy = (QL/k_f \Delta T)$, mean Nusselt number;</p> <p>Pr, $= \nu/a$, Prandtl number;</p> <p>$q(y')$, $= -k_f(\partial T/\partial n) _{x'=0}$, local heat flux at side wall;</p> <p>Q, $= \frac{1}{H} \int_0^H q(y') dy'$, mean heat flux at side wall;</p> <p>Ra, $= GrPr$, Rayleigh number;</p> <p>$T(x', y')$, air temperature;</p> <p>T_0, $= T(-\delta, y')$, temperature of the external surface of the left-hand thick wall of cavity;</p> <p>T_1, $= T(L, y')$, temperature of the right-hand wall of cavity;</p> <p>ΔT, $= T_1 - T_0$, maximum temperature difference in cavity;</p> <p>$u(x, y)$, $= \partial\psi/\partial y$ } horizontal and vertical $v(x, y)$, $= -\partial\psi/\partial x$ } velocity components;</p> <p>x, $= x'/L$ } dimensionless coordinates; y, $= y'/L$ } x', y', Cartesian coordinates;</p> <p>$\Delta x(\Delta y)$, grid spacing along $x(y)$.</p>	<p>β, coefficient of linear expansion of air;</p> <p>δ, thickness of side wall;</p> <p>$\theta(x, y)$, $= (T - T_0)/\Delta T$, dimensionless air temperature;</p> <p>ν, kinematic coefficient of viscosity;</p> <p>ψ, $= \psi'/\nu$, dimensionless stream function;</p> <p>$\psi'(x, y')$, stream function;</p> <p>$\omega(x', y')$, velocity vortex;</p> <p>Ω, $= L^2\omega/\nu$, dimensionless velocity vortex.</p>
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INTRODUCTION

THE PROBLEM of thermal conditions at the solid–fluid boundary in natural convection is among the particularly urgent ones [1]. Pre-assignment of temperature at the solid–fluid interface or heat flux through it is not always adequate. Thus, under the conditions of intensive heat transfer it fails to account for thermal interaction of the body with the surrounding non-isothermal fluid flow. Specification of a constant surface temperature in steady-state heat transfer is justifiable only in the case of high thermal conductivity of the body. It is, therefore, advisable to consider the problem as a conjugate one, i.e. to seek for a joint solution of equations of fluid convection and the equation of heat conduction in a body at equal *a priori* unknown temperatures and heat fluxes at the interface [2].

Very little research has been done to date on conjugate heat transfer natural convection for an interior problem. The solutions of two problems of steady-state heat transfer between a solid block and an incompressible fluid are given in the book by G. A. Ostroumov [3]: that of E. Drakhlin for the case of fluid filling a spherical region inside a block and that of E. M. Zhukhovitsky for the case of fluid filling an infinite horizontal channel in a block. In both cases, the method of a series expansion in a small non-linearity number Gr or Ra is used. The range of applicability of the method is established experimentally. No discussion is made of the effect of the boundary conditions of the fourth kind on heat transfer.

Greek symbols

α ,	$= (k_f/k_s)(\delta/L)$, parameter characterizing the ratio of solid wall to air thermal resistances;
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The steady-state convection of an incompressible fluid in a gap between two horizontal infinite coaxial cylinders was studied by Rotem [4]. The results of a numerical analysis of the solution, obtained in the form of the Stokes-type asymptotic expansions, are compared with the solution of the same problem in a non-conjugate formulation: the isotherm corresponding to $\theta = 0$ does not coincide any more with the core contour, as is the case for a non-conjugate problem, but penetrates into it; the isotherms are no longer concentric with the geometrical centre; in transition from the fluid into the core they experience a discontinuity which is greater, the smaller the ratio k_s/k_f .

Solution of the steady-state conjugate problem of natural convection in a gap between concentric conductive spheres, when the coefficient of thermal conductivity of the outer sphere is infinite and of the inner is finite, was obtained in the form of a double series in powers of Gr and Pr in [5].

Proof of the existence and uniqueness of the generalized solution to the problem of unsteady-state convection of viscous incompressible fluid in the field of body forces, when the fluid fills a cavity inside a body of finite dimensions, was given in [6].

Natural convection of electrically conductive fluid in a vertical channel was treated in [7]. The author isolated a region of parameters with an insignificant effect of the boundary conditions of the fourth kind on heat transfer which extends with increase in the ratio of the channel half-width to the electrical skin depth.

The general solution of the problem on fully developed laminar mixed convection in a vertical channel of arbitrary geometry with heat sources distributed uniformly in the wall of constant thickness was obtained by the variational method in [8]. The wall temperature, close to a constant one, is established at $K \rightarrow \infty$, where K is determined by the local Nusselt to Biot number ratio. At $K \rightarrow 0$, the heat flux at the wall becomes constant over the perimeter. With a substantial increase in the effect of natural convection and the conjugation parameter, K , the asymmetry in the wall temperature distribution over the perimeter decreases.

The general form of the conjugate generalized solution of an unsteady-state mass transfer problem in vertical plane porous-wall channels with concentration-induced convection of liquid (gas) was obtained by the method of generalized variables in [9]. This allowed the authors to process the experimental data on mass [10] and heat [11] transfer under the conditions of the problem stated.

The problem of conjugate heat transfer through a vertical porous layer bounded on one side by a plate of finite thickness, with the other being impenetrable or free, was solved in [12]. Investigation was carried out by numerical solution of full convective equations corresponding to a fine-dispersed isotropic porous medium and the linear filtration law [13]. At large Rayleigh numbers, in the approximation of the boundary layer, the solution was obtained for heat transfer

and temperature at the line of conjugation.

The present paper, which is a continuation of [14], is concerned with investigation of conjugate heat transfer through a plate of finite thickness and thermal conductivity and an air layer adjacent to it. The results of such a study may prove useful in the design calculations for thermal insulation elements of building constructions, in the search for optimum thermal operating conditions of the electronic equipment and so on.

STATEMENT OF THE PROBLEM

A geometry of the problem is given in Fig. 1. By the use of numerical simulation, a study is made of a two-dimensional steady-state problem of heat transfer through an air-filled vertical rectangular cavity with solid impenetrable walls. One of the side walls is a plate of finite thickness. The upper and lower bases of the cavity are thermally insulated, the vertical boundaries are kept at constant and different temperatures

$$T_0 \text{ at } x' = -\delta \quad \text{and} \quad T_1 \text{ at } x' = L \quad (T_0 \neq T_1).$$

To mathematically describe convective heat transfer in the cavity, use is made of the full system of convective equations in the Boussinesq approximation when the medium is assumed to be incompressible, and its thermophysical properties to be constant [15]. By eliminating pressure and incorporating the stream function, ψ , and vorticity, Ω , the system is brought to the form [16]

$$\frac{\partial}{\partial x} \left(\frac{\partial \Omega}{\partial x} - u\Omega \right) + \frac{\partial}{\partial y} \left(\frac{\partial \Omega}{\partial y} - v\Omega \right) + Gr \frac{\partial \theta}{\partial x} = 0 \quad (1)$$

$$\frac{\partial}{\partial x} \left(\frac{1}{Pr} \frac{\partial \theta}{\partial x} - u\theta \right) + \frac{\partial}{\partial y} \left(\frac{1}{Pr} \frac{\partial \theta}{\partial y} - v\theta \right) = 0 \quad (2)$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\Omega, \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \quad (3-5)$$

Here equations (1)–(5) are written in dimensionless form.

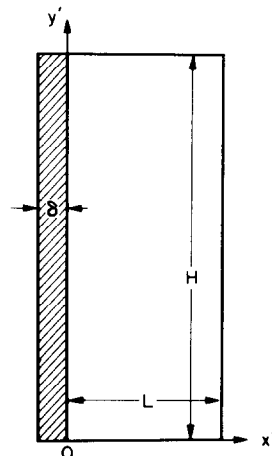


FIG. 1. Geometrical model.

In conformity with the statement of the problem, the conditions of immovability and impenetrability were prescribed at all the boundaries of the cavity

$$\psi = u = v = 0. \quad (6)$$

For the present investigation we shall confine our consideration to the case when the transverse thermal conductivity of the plate is much in excess of that in the direction y' . This allows the condition of equality of heat fluxes at the interface $x' = 0$ to be written as

$$k_f \cdot \frac{\partial T(0, y')}{\partial x'} = \frac{k_s}{\delta} [T(0, y') - T] \quad (7)$$

or in a dimensionless form

$$\alpha \cdot \frac{\partial \theta}{\partial x} = \theta. \quad (8)$$

Here $\alpha = (k_f/k_s)(\delta/L)$ is the ratio of the plate to the air layer thermal resistances.

At the other side boundary, $x = 1$, the condition of isothermicity was pre-assigned

$$\theta = \begin{cases} 1, & \text{when } T_0 < T_1 \\ -1, & \text{when } T_0 > T_1 \end{cases} \quad (9)$$

while at the horizontal boundaries of the cavity, $y = 0$ and $y = h$, the condition of thermal insulation was set

$$\frac{\partial \theta}{\partial y} = 0. \quad (10)$$

As for the boundary conditions for the velocity vortex, it was not possible to give their exact formulation. They were calculated approximately by the method described in [16].

METHOD OF SOLUTION

The system of equations (1)–(5) with boundary conditions (6)–(10) was solved numerically using a finite difference method. A monotonic conservative difference scheme of the second order of accuracy [16] was employed. The stationary solution was determined in the issue of Seidel's iteration. So that the convergence of iterations might be speeded up, equations for the stream function and velocity vortex incorporated relaxational parameters, the optimal value of which was determined and corrected in the course of the calculations. The calculations were carried out on a regular difference grid with spacings $\Delta x = 1/30$ and $\Delta y = 1/20$.

DISCUSSION OF RESULTS

The data, which will be given below, were obtained by analyzing numerical solutions of the system of equations (1)–(10) which correspond to the following ranges of operational parameters: $Pr = 0.72$; $Gr = 10^2$ – 10^7 ; $\alpha = 0$; 0.07; 0.11; 0.14; $h = 1$ – 10 . Two versions of the boundary conditions for the temperature at the wall, $x = 1$, were considered: the case of heating at this wall ($T_1 > T_0$) and the case of cooling ($T_1 < T_0$). Since the results for either case

are in the main similar, we shall confine our consideration for the convenience of the reader, to the case of $T_1 > T_0$ alone. The graphical material given below also refers to this case. Dimensional analysis of the problem considered shows that its complete solution is governed by the following functional relationship:

$$\left. \begin{matrix} \theta \\ \psi \end{matrix} \right\} = f(x, y, Ra, \alpha, h). \quad (11)$$

Although of no fundamental importance, it should be noted that, since all the calculations were carried out at one fixed value of $Pr = 0.72$, we shall speak below about the dependence of the solution on the Grashof number, Gr , rather than on the Rayleigh number, $Ra = PrGr$.

The characteristic features of the effect of Gr on thermoconvective processes in the cavity, revealed by the solution of the conjugate problem (1)–(10) at fixed values of other parameters, are close to the results obtained earlier from the solution of an analogous problem in non-conjugate formulation [17, 18], when both side boundaries of the layer were ideally conductive and isothermal. Increase in the Grashof number will cause intensification of convection, formation of the vertical temperature gradient and a change of heat transfer regimes in the cavity. The one close to a heat conduction regime ($Gr \leq 10^4$) is replaced by the boundary layer regime ($Gr \sim 10^5$) when the isotherms in the core are almost horizontal. A further increase in the Grashof number ($Gr \geq 10^6$) leads to a distortion of isotherms in the core and to the appearance of secondary circulating flows, a detailed description of which is given in [19].

The calculations show that a relative increase in thermal resistance of the plate in reference to thermal resistance of the air layer (i.e. increase of α), with temperature difference $\Delta T = T_1 - T_0$ being unchanged, corresponds to an appreciable decrease in the intensity of convective motions in the cavity. Thus, for example, at $\alpha \sim 0.1$ the maximum stream function characterizing the intensity of convection is almost 20% below that for the case $\alpha = 0$.

Decrease in the intensity of convective motion, observed with the rise of α , results in an elevation of temperature in the cavity. The main features of the effect of α on the structure of the temperature field in the layer are vividly illustrated in Fig. 2, which presents the distribution of temperature along the horizontal and vertical axes of the layer, as well as at the line of conjugation. The curves show that at all α s the air temperature at the conjugation line and close to it increases with the height of the layer. The pattern of its increase is close to a linear one. The temperature gradient hardly varies along the layer height. Its value increases with approach to the layer side boundaries and depends but slightly on α .

In consideration of heat transfer between a vertical air layer and a conductive plate with finite thickness, the local,

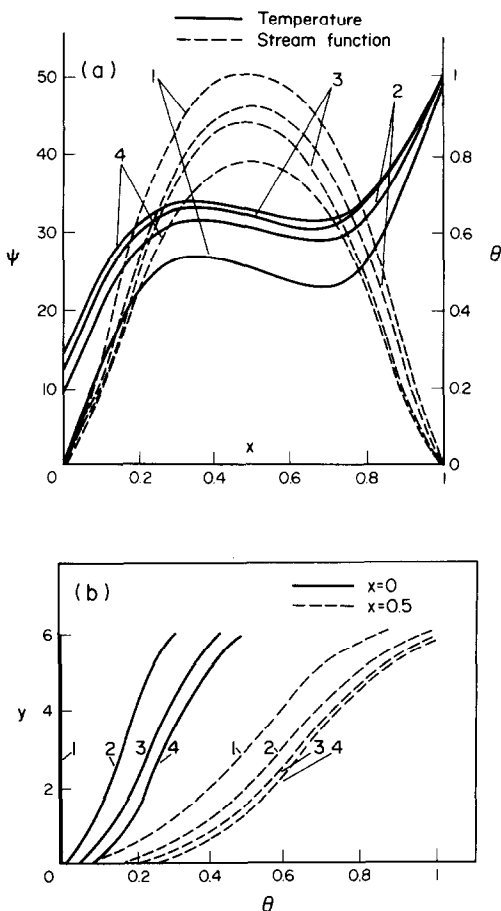


FIG. 2. Temperature and stream function distribution along the horizontal axis of the layer (a) and distribution of temperature along the vertical axis and the line of conjugation (b) at $\alpha = 0(1); 0.07(2); 0.11(3); 0.14(4)$; $Gr = 10^5$; $h = 6$.

$$q(y') = -k_f \frac{\partial T}{\partial n} \Big|_{x'=0},$$

and average,

$$Q = \frac{1}{H} \int_0^H q(y') dy',$$

heat fluxes at the conjugation line, which are of the greatest interest for application, will be given in terms of the local and average Nusselt numbers

$$Nu_t = \frac{qL}{k_f \Delta T} = - \frac{\partial \theta}{\partial n} \Big|_{x=0},$$

$$Nu = - \frac{1}{h} \int_0^h \frac{\partial \theta}{\partial n} \Big|_{x=0} dy = \frac{QL}{k_f \Delta T}. \quad (12)$$

It has been shown in the works of A. V. Luikov [20, 21] that over the range of the Rayleigh numbers $10^2 \leq Ra \leq 10^7$, the mutual effect of the temperature fields of the wall and fluid flow past it under the

conditions of natural convection may be characterized by Brun's dimensionless parameter

$$Br = \frac{k_f}{k_s} \cdot \frac{\delta}{y} \cdot Ra_y^{1/4} \quad (13)$$

which is the value proportional to the ratio between the wall and the fluid thermal resistances.

In the course of analysis and processing of the results obtained it has become possible to construct a single-valued dependence of Nu/Nu_0 on $Br_L = \alpha Ra^{1/4}$, where Nu_0 is the average Nusselt number in a layer with ideally conductive isothermal boundaries ($Br_L = 0$). In this case, the scatter in the values of Nu/Nu_0 at fixed Br_L did not exceed 10–20%, while the value of Nu itself varied several-fold over the range of parameters considered.

The curve presented in Fig. 3 makes it possible to calculate the average Nusselt number for the conjugate heat transfer when the average Nusselt number for the layer with ideally conductive isothermal boundaries is known; while, for determining the latter, there are many approximate formulae obtained by a number of authors [22, 23].

Processing of the obtained numerical results by the least square method has made it possible, in the range $5 \times 10^3 \leq Gr \leq 5 \times 10^6$, to construct, for different values of α , the power dependences of the average Nusselt number on Gr and h

$$Nu = 0.225 Gr^{0.25} h^{-0.12}, \quad (\alpha = 0) \quad (14)$$

$$Nu = 0.276 Gr^{0.196} h^{-0.12}, \quad (\alpha = 0.07) \quad (15)$$

$$Nu = 0.334 Gr^{0.174} h^{-0.12}, \quad (\alpha = 0.14). \quad (16)$$

Note that the average Nusselt numbers calculated by formula (14) and corresponding to the layer with isothermal and ideally conductive boundaries, are 8–10% below and 9–12% above the values of Nu calculated respectively from the formulae of Elder [22] and Jakob [23].

Inspection of the distribution of the local Nusselt numbers shows that at low Grashof numbers ($Gr \leq 5 \times 10^3$) the local heat fluxes on the side boundaries of the layer practically remain unchanged with the height of the layer, the only exception being small segments close to the base of the layer. Here, an increase in α leads to a small overall decrease in the local Nusselt numbers.

At $Gr \sim 10^4$ and above, due to the effect of the natural convection, non-uniformity in distribution of the local Nusselt numbers over the vertical boundaries of the layer becomes more appreciable. In the vicinity of the upper portion of the thick plate, the absolute values of Nu_t are the highest. With downward fluid flow along the plate, the values of Nu_t decrease rather rapidly. Rise in thermal resistance of the plate at fixed Gr leads to a decrease in the local heat removal due to a drop in the intensity of convection. A similar pattern is also observed on the right-hand thin boundary of the layer with the only difference that the values of Nu_t are

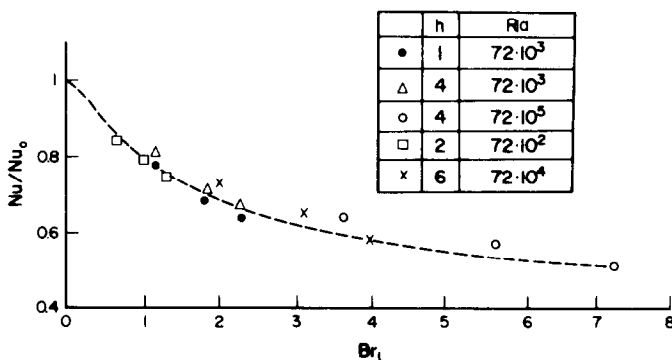


FIG. 3. Average Nusselt number for conjugate heat transfer.

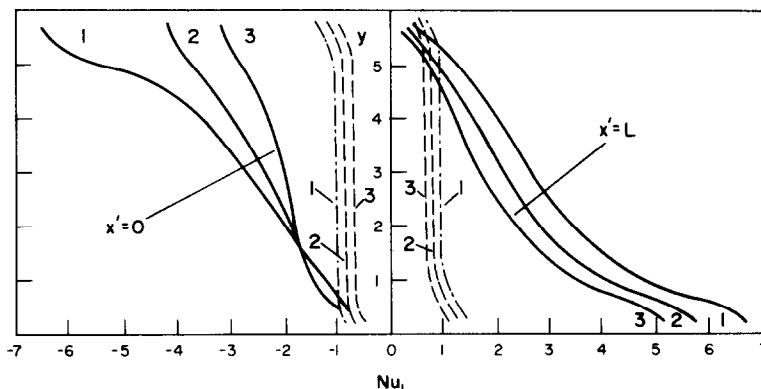


FIG. 4. Distribution of the local Nusselt number along the layer side boundaries with $h = 6$ at $Gr = 10^3$ (broken curves), $Gr = 10^5$ (solid curves) and $\alpha = 0$ (1); 0.7 (2); 0.14 (3).

highest in its lower portion and decrease with an increase in height, which is attributed to the structure of convective motion in the cavity (Fig. 4).

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EFFET DES PAROIS LATÉRALES SUR LE TRANSFERT THERMIQUE
A TRAVERS UNE COUCHE D'AIR VERTICALE EN CONVECTION
LAMINAIRE NATURELLE

Résumé — On présente les résultats de l'étude numérique du transfert thermique local et global d'une couche d'air verticalement limitée, d'épaisseur finie et de conductivité thermique donnée. On discute l'effet du rapport des résistances thermiques de la paroi et de l'air sur le transfert thermique. L'étude concerne les domaines suivants de variation des paramètres $Pr = 0,72$; $Gr < 10^7$; $1 < h < 10$; $0 < \alpha < 0,14$.

EINFLUß DER SEITENWÄNDE AUF DEN WÄRMETRANSPORT BEI FREIER
KONVEKTION DURCH EINE VERTIKALE LUFTSCHICHT

Zusammenfassung — Der Bericht beschreibt die Ergebnisse der numerischen Untersuchung des lokalen und mittleren Wärmetransports in einer vertikal begrenzten Luftschicht, deren eine seitliche Begrenzung endliche Dicke und Wärmeleitfähigkeit besitzt. Der Einfluß des Verhältnisses vom Wärmeleitwiderstand der Wand zu dem der Luft auf den Wärmetransport wird diskutiert. Die Untersuchung wurde für den folgenden Parameterbereich durchgeführt: $Pr = 0,72$; $Gr < 10^7$; $1 \leq h \leq 10$; $0 \leq \alpha \leq 0,14$.

ВЛИЯНИЕ БОКОВЫХ СТЕНОК НА ТЕПЛООБМЕН ЧЕРЕЗ ВЕРТИКАЛЬНЫЙ СЛОЙ
ВОЗДУХА ПРИ ЛАМИНАРНОЙ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ

Аннотация — Излагаются результаты численного исследования локального и среднего теплообмена вертикального ограниченного слоя воздуха, одна из боковых границ которого имеет конечную толщину и теплопроводность. Обсуждается влияние относительной величины термических сопротивлений стенки и жидкости на теплообмен. Исследования проведены в диапазоне параметров: $Pr = 0,72$; $Gr < 10^7$; $1 \leq h \leq 10$; $0 \leq \alpha \leq 0,14$.